2.49

2.49 A rectangular gate having a width of 5 ft is located in the sloping side of a tank as shown in Fig. P2.49. The gate is hinged along its top edge and is held in position by the force P. Friction at the hinge and the weight of the gate can be neglected. Determine the required value of P.

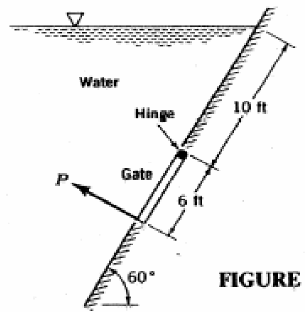


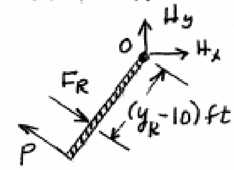
FIGURE P2.49

$F_R = \gamma h_c A$ where $h_c = (13\ ft)\sin 60°$

Thus,

$$F_R = \left(62.4\ \frac{lb}{ft^3}\right)(13\ ft)\sin 60° (6\ ft \times 5 ft)$$

$$= 21,100\ lb$$



Also,

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(5\ ft)(6 ft)^3}{(13\ ft)(6 ft \times 5 ft)} + 13\ ft = 13.23\ ft$$

$\Sigma M_o = 0$

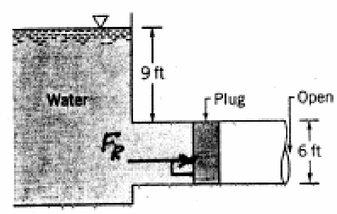
Thus, $F_R\left[(y_R - 10)\ ft\right] = P\ (6\ ft)$

so that

$$P = \frac{(21,100\ lb)(13.23\ ft - 10 ft)}{6\ ft} = \underline{\underline{11,400\ lb}}$$

2.51

2.51 A large, open tank contains water and is connected to a 6-ft diameter conduit as shown in Fig. P2.51. A circular plug is used to seal the conduit. Determine the magnitude, direction, and location of the force of the water on the plug.



■ **FIGURE P2.51**

$$F_R = \gamma h_c A = \left(62.4\ \frac{lb}{ft^3}\right)(12\ ft)\left(\frac{\pi}{4}\right)(6\ ft)^2 = \underline{\underline{21,200\ lb}}$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c \quad \text{where} \quad I_{xc} = \frac{\pi\ (3ft)^4}{4} = 63.6\ ft^4$$

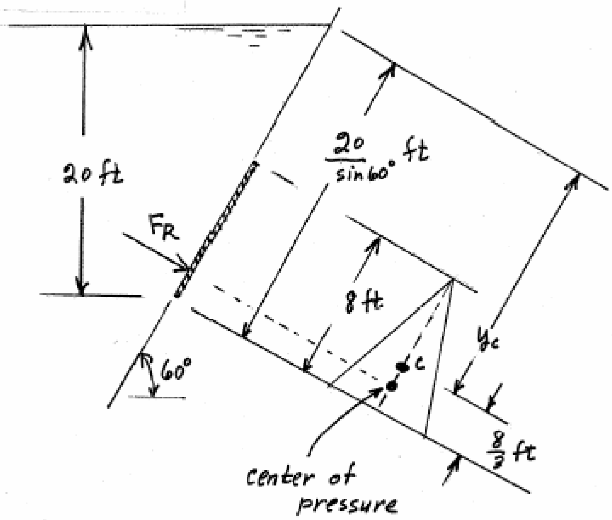
Thus,

$$y_R = \frac{\frac{\pi}{4}(3 ft)^4}{(12\ ft)\pi\ (3ft)^2} + 12\ ft = \underline{\underline{12.19\ ft}}$$

The force of 21,200 lb acts 12.19 ft below the water surface and is perpendicular to the plug surface as shown.

2.54

2.54 An area in the form of an isosceles triangle with a base width of 6 ft and an altitude of 8 ft lies in the plane forming one wall of a tank which contains a liquid having a specific weight of 79.8 lb/ft³. The side slopes upward making an angle of 60° with the horizontal. The base of the triangle is horizontal and the vertex is above the base. Determine the resultant force the fluid exerts on the area when the fluid depth is 20 ft above the base of the triangular area. Show, with the aid of a sketch, where the center of pressure is located.



$$y_c = \left(\frac{20}{\sin 60°}\right) ft - \left(\frac{8}{3}\right) ft$$

$$= 20.43 \ ft$$

$$h_c = y_c \sin 60°$$

$$F_R = \gamma h_c A = \left(79.8 \frac{lb}{ft^3}\right)\left[(20.43 \ ft) \sin 60°\right]\left(\frac{1}{2}\right)(6 ft \times 8 ft)$$

$$= \underline{33,900 \ lb}$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c \qquad \text{where} \quad I_{xc} = \frac{1}{36}(6ft)(8ft)^3$$

Thus,

$$y_R = \frac{\frac{1}{36}(6ft)(8ft)^3}{(20.43ft)(\frac{1}{2})(6ft \times 8ft)} + 20.43 \ ft = 20.6 \ ft$$

The force, F_R, acts through the center of pressure which is located a distance of $\frac{20}{\sin 60°}$ ft - 20.6 ft = $\underline{2.49 \ ft}$ above the base of the triangle as shown in sketch.

2.58

2.58 A gate having the cross section shown in Fig. P2.58 closes an opening 5 ft wide and 4 ft high in a water reservoir. The gate weighs 500 lb and its center of gravity is 1 ft to the left of AC and 2 ft above BC. Determine the horizontal reaction that is developed on the gate at C.

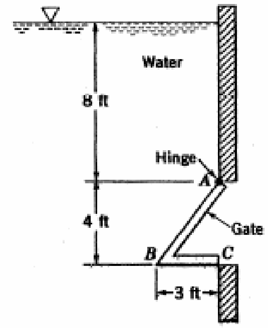


FIGURE P2.58

$F_1 = \gamma h_{c1} A_1$ where $h_{c1} = 8\,ft + 2\,ft$

Thus,

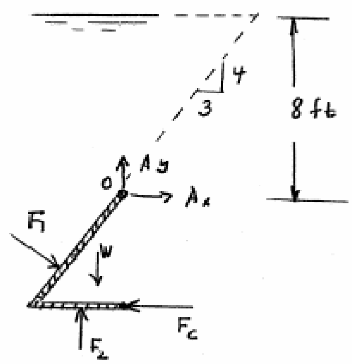
$F_1 = (62.4 \frac{lb}{ft^3})(10\,ft)(5\,ft \times 5\,ft)$

$= 15,600\ lb$

To locate F_1,

$$y_1 = \frac{I_{xc}}{y_{c1} A_1} + y_{c1}$$

where $y_{c1} = \frac{8\,ft}{\frac{4}{5}} + 2.5\,ft = 12.5\,ft$



So that

$$y_1 = \frac{\frac{1}{12}(5\,ft)(5\,ft)^3}{(12.5\,ft)(5\,ft \times 5\,ft)} + 12.5\,ft = 12.67\,ft$$

Also,

$F_2 = p_2 A_2$ where $p_2 = \gamma_{H_2O}(8\,ft + 4\,ft)$

so that

$F_2 = \gamma_{H_2O}(12\,ft)(A_2) = (62.4 \frac{lb}{ft^3})(12\,ft)(3\,ft \times 5\,ft) = 11,230\ lb$

For equilibrium,

$\sum M_o = 0$

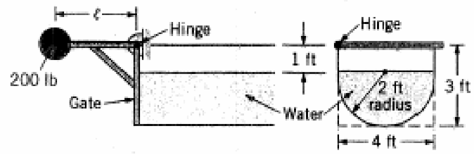
and $F_1 \left(y_1 - \frac{8\,ft}{\frac{4}{5}}\right) + W(1\,ft) - F_2(\tfrac{1}{2})(3\,ft) - F_c(4\,ft)$

so that

$$F_c = \frac{(15,600\,lb)(12.67\,ft - 10\,ft) + (500\,lb)(1\,ft) - (11,230\,lb)(\frac{3}{2}\,ft)}{4\,ft} = \underline{\underline{6330\,lb}}$$

2.62

2.62 A 4-ft by 3-ft massless rectangular gate is used to close the end of the water tank shown in Fig. P2.62. A 200 lb weight attached to the arm of the gate at a distance ℓ from the frictionless hinge is just sufficient to keep the gate closed when the water depth is 2 ft, that is, when the water fills the semicircular lower portion of the tank. If the water were deeper the gate would open. Determine the distance ℓ.



■ FIGURE P2.62

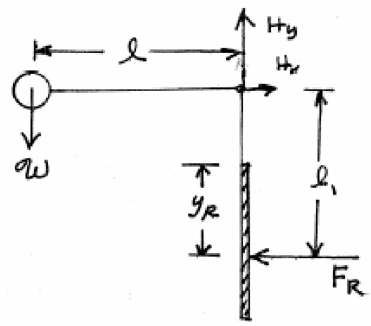
$F_R = \gamma h_c A$ where $h_c = \frac{4R}{3\pi}$ (see Fig. 2.18)

Thus,

$$F_R = \gamma_{H_2O}\left(\frac{4R}{3\pi}\right)\left(\frac{\pi R^2}{2}\right)$$

$$= \left(62.4\,\frac{lb}{ft^3}\right)\left(\frac{4\,(2ft)}{3\pi}\right)\left(\pi\,\frac{(2ft)^2}{2}\right)$$

$$= 333\ lb$$



To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

$$= \frac{0.1098\,R^4}{\left(\frac{4R}{3\pi}\right)\left(\frac{\pi R^2}{2}\right)} + \frac{4R}{3\pi} \quad \text{(see Fig. 2.18)}$$

$\ell_1 = 1\,ft + y_R$

$$= \frac{(0.1098)(2ft)^4}{\left(\frac{4\,(2ft)}{3\pi}\right)\frac{\pi(2ft)^2}{2}} + \frac{4\,(2ft)}{3\pi} = 1.178\ ft$$

For equilibrium,

$$\Sigma M_H = 0$$

so that

$$\mathcal{W}\ell = F_R\,(1\,ft + y_R)$$

and

$$\ell = \frac{(333\,lb)(1\,ft + 1.178\,ft)}{200\,lb} = \underline{\underline{3.63\ ft}}$$

2.67

2.67 The closed vessel of Fig. P2.67 contains water with an air pressure of 10 psi at the water surface. One side of the vessel contains a spout that is closed by a 6-in.-diameter circular gate that is hinged along one side as illustrated. The horizontal axis of the hinge is located 10 ft below the water surface. Determine the minimum torque that must be applied at the hinge to hold the gate shut. Neglect the weight of the gate and friction at the hinge.

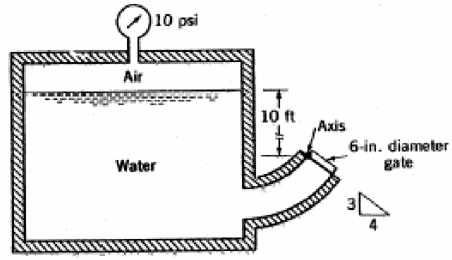
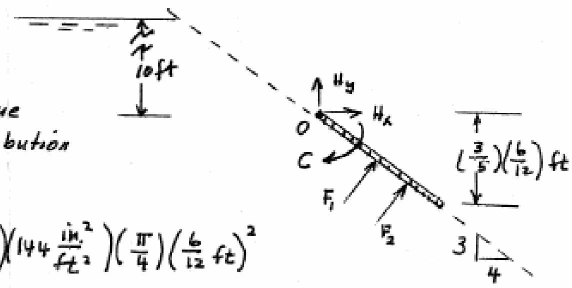


FIGURE P2.67

Let $F_1 \sim$ force due to air pressure, and $F_2 \sim$ force due to hydrostatic pressure distribution of water.



Thus,

$$F_1 = p_{air} A = \left(10\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)\left(\frac{\pi}{4}\right)\left(\frac{6}{12}\ ft\right)^2$$

$$= 283\ lb$$

and

$$F_2 = \gamma h_c A \quad \text{where} \quad h_c = 10\ ft + \frac{1}{2}\left[\left(\frac{3}{5}\right)\left(\frac{6}{12}\right) ft\right] = 10.15\ ft$$

so that

$$F_2 = \left(62.4\ \frac{lb}{ft^3}\right)(10.15\ ft)\left(\frac{\pi}{4}\right)\left(\frac{6}{12}\ ft\right)^2 = 124\ lb$$

Also,

$$y_{R2} = \frac{I_{xc}}{y_c A} + y_c \quad \text{where} \quad y_c = \frac{10\ ft}{\frac{3}{5}} + \frac{1}{2}\left(\frac{6}{12}\ ft\right) = 16.92\ ft$$

so that

$$y_{R2} = \frac{\left(\frac{\pi}{4}\right)\left(\frac{3}{12}\ ft\right)^4}{(16.92\ ft)\left(\frac{\pi}{4}\right)\left(\frac{6}{12}\ ft\right)^2} + 16.92\ ft = 16.92\ ft$$

For equilibrium,

$$\sum M_o = 0$$

and

$$C = F_1\left(\frac{3}{12}\ ft\right) + F_2\left(y_{R2} - \frac{10\ ft}{\frac{3}{5}}\right)$$

or

$$C = (283\ lb)\left(\frac{3}{12}\ ft\right) + (124\ lb)\left(16.92\ ft - \frac{10\ ft}{\frac{3}{5}}\right) = \underline{\underline{102\ ft \cdot lb}}$$

2.68

2.68 Dams can vary from very large structures with curved faces holding back water to great depths, as shown in Video V2.3, to relatively small structures with plane faces as shown in Fig. P2.68. Assume that the concrete dam shown in Fig. P2.68 weighs 23.6 kN/m^3 and rests on a solid foundation. Determine the minimum coefficient of friction between the dam and the foundation required to keep the dam from sliding at the water depth shown. You do not need to consider possible uplift along the base. Base your analysis on a unit length of the dam.

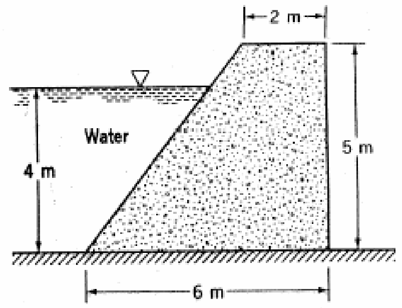
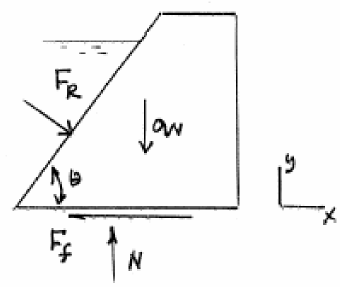


FIGURE P2.68



$$F_R = \gamma h_c A$$

where $A = \left(\frac{4\ m}{\sin 51.3°}\right)(1\ \ )$

so that

$$F_R = \left(9.80 \frac{kN}{m^3}\right)\left(\frac{4\,m}{2}\right)\left(\frac{4\ m}{\sin 51.3°}\right)(1\,m)$$
$$= 100\ kN$$

$\tan\theta = \frac{5\ m}{4\ m}$

$\theta = 51.3°$

For equilibrium,

$$\Sigma F_x = 0$$

or

$$F_R \sin 51.3° = F_f = \eta N$$ where $\eta \sim$ coefficient of friction.

Also, $\Sigma F_y = 0$

so that

$$N = \mathcal{W} + F_R \cos 51.3°$$ where

$$\mathcal{W} = (\gamma_{concrete})(\text{volume of concrete})$$

Thus,

$$N = \left(23.6\ \frac{kN}{m^3}\right)(20\ m^3) + (100\,kN)\cos 51.3° = 534\ kN$$

and

$$\eta = \frac{F_R \sin 51.3°}{N} = \frac{(100\,kN)\sin 51.3°}{534\ kN} = \underline{\underline{0.146}}$$

2.52

2.52 A homogeneous, 4-ft-wide, 8-ft-long rectangular gate weighing 800 lb is held in place by a horizontal flexible cable as shown in Fig. P2.52. Water acts against the gate which is hinged at point A. Friction in the hinge is negligible. Determine the tension in the cable.

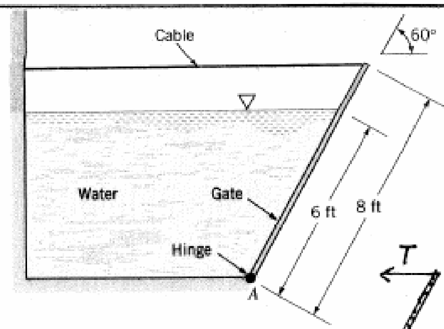
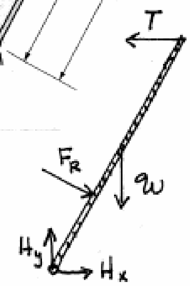


FIGURE P2.52



$$F_R = \gamma h_c A \quad \text{where} \quad h_c = \left(\frac{6\,ft}{2}\right)\sin 60^\circ$$

Thus,

$$F_R = \left(62.4\,\frac{lb}{ft^3}\right)\left(\frac{6\,ft}{2}\right)(\sin 60^\circ)(6\,ft \times 4\,ft)$$

$$= 3890\ lb$$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c \qquad \text{where} \quad y_c = 3\,ft$$

so that

$$y_R = \frac{\frac{1}{12}(4\,ft)(6\,ft)^3}{(3\,ft)(6\,ft \times 4\,ft)} + 3\,ft = 4.0\ ft$$

For equilibrium,

$$\Sigma M_H = 0$$

and

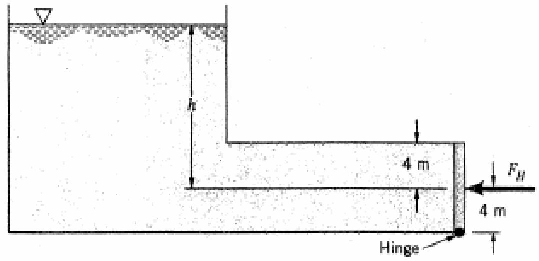
$$T\,(8\,ft)(\sin 60^\circ) = \mathcal{W}\,(4\,ft)(\cos 60^\circ) + F_R\ (2\,ft)$$

$$T = \frac{(800\,lb)(4\,ft)(\cos 60^\circ) + (3890\,lb)(2\,ft)}{(8\,ft)(\sin 60^\circ)}$$

$$= \underline{\underline{1350\ lb}}$$

2.57

2.57 A 3-m-wide, 8-m-high rectangular gate is located at the end of a rectangular passage that is connected to a large open tank filled with water as shown in Fig. P2.57. The gate is hinged at its bottom and held closed by a horizontal force, F_H, located at the center of the gate. The maximum value for F_H is 3500 kN. **(a)** Determine the maximum water depth, h, above the center of the gate that can exist without the gate opening. **(b)** Is the answer the same if the gate is hinged at the top? Explain your answer.



■ FIGURE P2.57

For gate hinged at <u>bottom</u>

$$\sum M_H = 0$$

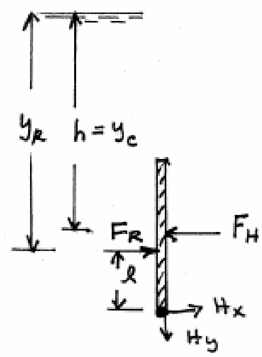
so that

$$(4m)\,F_H = \ell\,F_R \quad (\text{see figure}) \quad (1)$$

and

$$F_R = \gamma h_c A = \left(9.80\,\frac{kN}{m^3}\right)(h)(3m \times 8m)$$

$$= (9.80 \times 24\,h)\,kN$$



$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(3m)(8m)^3}{h\,(3m \times 8m)} + h$$

$$\ell = h + 4 - y_R$$

$$= \frac{5.33}{h} + h$$

Thus,

$$\ell(m) = h + 4 - \left(\frac{5.33}{h} + h\right) = 4 - \frac{5.33}{h}$$

and from Eq. (1)

$$(4\,m)(3500\,kN) = \left(4 - \frac{5.33}{h}\right)(9.80 \times 24)(h)\;kN$$

so that

$$\underline{\underline{h = 16.2\ m}}$$

(con't)

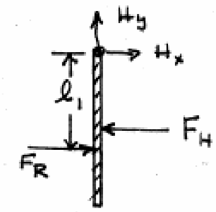
2.57 (Con't)

For gate hinged at <u>top</u>

$\sum M_H = 0$

so that

$(4\,m) F_H = \ell_1 F_R$ (see figure) (1)



$\ell_1 = y_R - (h-4)$

where

$$\ell_1 = y_R - (h-4) = \left(\frac{5.33}{h} + h\right) - (h-4)$$

$$= \frac{5.33}{h} + 4$$

Thus, from Eq. (1)

$$(4\,m)(3500\,kN) = \left(\frac{5.33}{h} + 4\right)(9.80 \times 24)(h)\ kN$$

and

$$\underline{\underline{h = 13.5\,m}}$$

Maximum depth for gate hinged at top is less than maximum depth for gate hinged at bottom.

2.59

2.59 The massless, 4-ft-wide gate shown in Fig. P2.59 pivots about the frictionless hinge O. It is held in place by the 2000 lb counterweight, W. Determine the water depth, h.

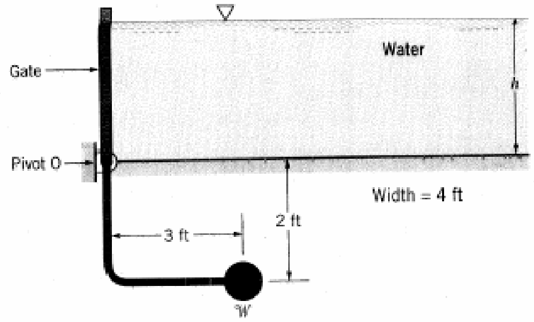
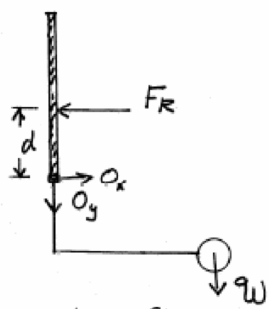


FIGURE P2.59

$F_R = \gamma h_c A$ where $h_c = \frac{h}{2}$



Thus,

$$F_R = \gamma_{H_2O} \frac{h}{2} (h \times b)$$

$$= \gamma_{H_2O} \frac{h^2}{2} (4 ft)$$

$b = 4 ft$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(4 ft)(h^3)}{\frac{h}{2}(4ft \times h)} + \frac{h}{2}$$

$$= \frac{2}{3} h$$

For equilibrium,

$$\Sigma M_o = 0$$

$$F_R \, d = W (3 ft) \quad \text{where} \quad d = h - y_R = \frac{h}{3}$$

so that

$$\frac{h}{3} = \frac{(2000 \, lb)(3 ft)}{(\gamma_{H_2O})(\frac{h^2}{2})(4 ft)}$$

Thus,

$$h^3 = \frac{(3)(2000 \, lb)(3 ft)}{(62.4 \frac{lb}{ft^3})(\frac{1}{2})(4 ft)}$$

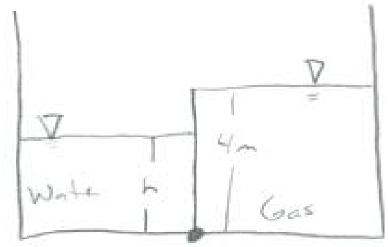
$$h = \underline{\underline{5.24 ft}}$$

2.61)

Gas $\rho = 700 \frac{kg}{m^3}$

$\gamma = \rho g = 700(9.81)$

$= 6.867 \ KN/m^3$



$F_{R_{Gas}} = \gamma h_c * A = 6.867(2)\,4(2) = 109.87 \ KN$

since rectangle shape,

$y_R = 2.67m$ from top

$F_{R_{water}} = \gamma h_c A = 9.8\left(\frac{h}{\cancel{2}}\right) h(\cancel{2}) = 9.8h^2$

$y_R = \frac{2}{3}h$ from top of water

$\Sigma M_{Hinge} = 0$ (↻+)

$F_{R_{Gas}}(4 - y_R) - F_{R_{water}}(h - \frac{2}{3}h) = 0$

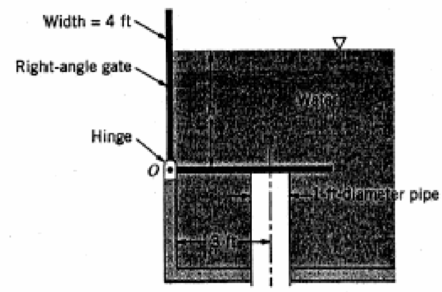
$109.87(4 - 2.67) = 9.8h^2\left(\frac{1}{3}h\right)$

$44.844 = h^3$

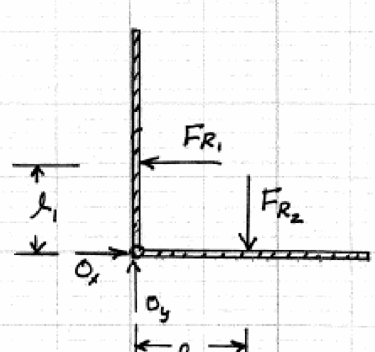
$\therefore \ \boxed{h = 3.55m}$

2.64

2.64 A thin 4-ft-wide, right-angle gate with negligible mass is free to pivot about a frictionless hinge at point O, as shown in Fig. P2.64. The horizontal portion of the gate covers a 1-ft-diameter drain pipe which contains air at atmospheric pressure. Determine the minimum water depth, h, at which the gate will pivot to allow water to flow into the pipe.



■ FIGURE P2.64



For equilibrium

$$\sum M_0 = 0$$

$$F_{R_1} \times \ell_1 = F_{R_2} \times \ell_2 \qquad (1)$$

$$F_{R_1} = \gamma h_{c_1} A_1$$
$$= \left(62.4 \tfrac{lb}{ft^3}\right)\left(\tfrac{h}{2}\right)(4ft \times h)$$
$$= 125\, h^2$$

For the force on the horizontal portion of the gate (which is balanced by pressure on both sides except for the area of the pipe)

$$F_{R_2} = \gamma h \left(\tfrac{\pi}{4}\right)(1ft)^2 = \left(62.4 \tfrac{lb}{ft^3}\right)(h)\left(\tfrac{\pi}{4}\right)(1ft)^2$$
$$= 49.0\, h$$

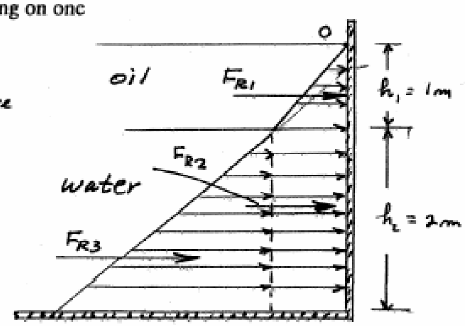
Thus, from Eq. (1) with $\ell_1 = \frac{h}{3}$ and $\ell_2 = 3\,ft$

$$(125 h^2)\left(\tfrac{h}{3}\right) = (49.0\, h)(3 ft)$$

$$h = \underline{\underline{1.88\, ft}}$$

2.65

2.65 An open rectangular tank is 2 m wide and 4 m long. The tank contains water to a depth of 2 m and oil ($SG = 0.8$) on top of the water to a depth of 1 m. Determine the magnitude and location of the resultant fluid force acting on one end of the tank.



Use the concept of the pressure prism (see figure).

$$F_{R1} = \gamma_{oil}\, h_{c1}\, A_1$$

so that

$$F_{R1} = (0.8)\left(9.81\,\frac{kN}{m^3}\right)\left(\frac{1m}{2}\right)(1m \times 2m)$$
$$= 7.85\ kN$$

Let $w \sim$ width $= 2m$

$F_{R2} = p_2 A_2$ where p_2 is pressure at depth h_1. Thus,

$$F_{R2} = (\gamma_{oil}\, h_1)(h_2 \times w) = (0.8)\left(9.81\,\frac{kN}{m^3}\right)(1m)(2m \times 2m) = 31.4\ kN$$

Also,

$$F_{R3} = \gamma_{H_2O}\, h_{c3}\, A_3 \qquad \text{so that}$$

$$F_{R3} = \gamma_{H_2O}\left(\frac{h_2}{2}\right)(h_2 \times w) = \left(9.80\,\frac{kN}{m^3}\right)\left(\frac{2m}{2}\right)(2m \times 2m) = 39.2\ kN$$

Thus,

$$F_R = F_{R1} + F_{R2} + F_{R3} = 7.85\,kN + 31.4\,kN + 39.2\,kN = \underline{\underline{78.5\,kN}}$$

To locate F_R sum moments around axis through O, so that

$$F_R\, d_R = F_{R1}\, d_1 + F_{R2}\, d_2 + F_{R3}\, d_3 \qquad (1)$$

where d_R is distance to F_R. Since F_{R1}, F_{R2}, and F_{R3} act through the centroids of their respective pressure prisms it follows that

$$d_1 = \tfrac{2}{3}(1m) \quad , \quad d_2 = 1m + 1m = 2m, \quad d_3 = 1m + \tfrac{2}{3}(2m)$$

and from Eq.(1)

$$d = \frac{(7.85\,kN)\left(\frac{2}{3}\right)(1m) + (31.4\,kN)(2m) + (39.2\,kN)\left(1m + \frac{4m}{3}\right)}{78.5\,kN}$$

$$= \underline{\underline{2.03\ m \quad (\text{below oil free surface})}}$$